KINEMATICS

*rest and Motion are relative terms, nobody can exist in a state of absolute rest or of absolute motion.

*One dimensional motion:- The motion of an object is said to be one dimensional motion if only one out of three coordinates specifying the position of the object change with time. In such a motion an object move along a straight line path.

*Two dimensional motion:- The motion of an object is said to be two dimensional motion if two out of three coordinates specifying the position of the object change with time. In such motion the object moves in a plane.

*Three dimensional motion:- The motion is said to be three dimensional motion if all the three coordinates specifying the position of an object change with respect to time ,in such a motion an object moves in space.

*The magnitude of displacement is less than or equal to the actual distance travelled by the object in the given time interval.

$$\text{Displacement} \leq \text{Actual distance}$$

*Speed:- It is rate of change of distance covered by the body with respect to time.

$$\text{Speed} = \text{Distance travelled} / \text{time taken}$$

Speed is a scalar quantity .Its unit is meter /sec. and dimensional formula is $[M^0L^1T^{-1}]$ .It is positive or zero but never negative.

*Uniform Speed:- If an object covers equal distances in equal intervals of time than the speed of the moving object is called uniform speed. In this type of motion, position – time graph is always a straight line.

*Instantaneous speed:-The speed of an object at any particular instant of time is called instantaneous speed. In this measurement, the time $\Delta t \rightarrow 0$.

When a body is moving with uniform speed its instantaneous speed = Average speed = uniform speed.

*Velocity:- The rate of change of position of an object in a particular direction with respect to time is called velocity. It is equal to the displacement covered by an object per unit time.

Velocity =Displacement /Time

Velocity is a vector quantity, its SI unit is meter per sec. Its dimensional formula is $[M^0L^1T^{-1}]$. It may be negative, positive or zero.

*When a body moves in a straight line then the average speed and average velocity are equal.

*Acceleration:- The rate of change of velocity of an object with respect to time is called its acceleration.

Acceleration = Change in velocity /time taken

It is a vector quantity, Its SI unit is meter/ $sec.^2$ and dimension is $[M^0L^1T^{-2}]$, It may be positive ,negative or zero.

*Positive Acceleration:- If the velocity of an object increases with time, its acceleration is positive .

*Negative Acceleration :-If the velocity of an object decreases with time, its acceleration is negative . The negative acceleration is also called retardation or deacceleration.

*Formulas of uniformly accelerated motion along straight line:-

For accelerated motion,	For deceleration motion
$V = u + at$	$v = u - at$
$S = ut + {}^1/_2\ at^2$	$S = ut - {}^1/_2\ at^2$
$V^2 = u^2 + 2as$	$V^2 = u^2 - 2as$
$Sn = u + {}^a/_2(2n- 1)$	$Sn = u - {}^a/_2(2n- 1)$

*Free fall :- In the absence of the air resistance all bodies fall with the same acceleration towards earth from a small height. This is called free fall. The acceleration with which a body falls is called gravitational acceleration (g).Its value is 9.8 m/sec^2.

*Relative Motion:- The rate of change of distance of one object with respect to the other is called relative velocity. The relative velocity of an object B with respect to the object A when both are in motion is the rate of change of position of object B with respect to the object A.

*Relative velocity of object A with respect to object B

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

When both objects are move in same direction, then the relative velocity of object B with respect to the object A

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

When the object B moves in opposite direction of object A .

$$\vec{V}_{BA} = \vec{V}_B + \vec{V}_A$$

When V_A and V_B are incident to each other at angle Θ

$$V_{AB} = (V^2_A + V^2_B - 2V_AV_BCos\ \Theta)^{1/2}$$

*Scalars :- The quantities which have magnitude only but no direction. For example : mass, length, time, speed , temperature etc.

*Vectors :- The quantities which have magnitude as well as direction and obeys vector laws of addition, multiplication etc.

For examples : Displacement, velocity, acceleration, force , momentum etc.

- **Addition of Vectors :-**
  - (i) Only vectors of same nature can be added.
  - (ii) The addition of two vector A and B is resultant R

$$\vec{R} = \vec{A} + \vec{B}$$

And $R = (A^2 + B^2 + 2AB \cos\Theta)^{1/2}$

And $\tan\beta = B \sin\Theta / (A + B \cos\Theta)$,

Where Θ is the angle between vector A and vector B, And β is the angle which R makes with the direction of A.

(iii) Vector addition is commutative $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

(iv) Vector addition is associative,

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

(v) R is maximum if Θ = 0 and minimum if Θ = 180 ^{0}.

Subtraction of two vectors :-

(i) Only vector of same nature can be subtracted.

(ii) Subtraction of B from A = vector addition of A and (-B),

$$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (\overrightarrow{-B})$$

Where $R = [A^2 + B^2 + 2AB \cos(180 - \Theta)]^{1/2}$ and

$\tan\beta = B \sin(180 - \Theta) / [A + B \cos(180 - \Theta)]$, Where Θ is the angle between A and B and β is the angle which R makes with the direction of A.

(iii) Vector subtraction is not commutative $(\overrightarrow{A} - \vec{B}) \neq (\overrightarrow{B} - \vec{A})$

(iv) Vector subtraction is not associative,

$$(\vec{A} - \vec{B}) - \vec{C} \neq \vec{A} - (\vec{B} - \vec{C})$$

Rectangular components of a vector in a plane :- If A makes an angle Θ with x-axis and A_x and B_y be the rectangular components of A along X-axis and Y- axis respectively, then

$$\vec{A} = \vec{A}_x + \vec{B}_y = A_x \hat{i} + A_y \hat{j}$$

Here $A_x = A\ Cos\Theta$ and $A_y = A\ Sin\Theta$

And $A = (A_x^2 + A_y^2)^{1/2}$

And $\tan\Theta = A_y/A_x$

Dot product or scalar product : - The dot product of two vectors A and B, represented by $\vec{A}.\vec{B}$ is a scalar , which is equal to the product of the magnitudes of A and B and the Cosine of the smaller angle between them.

If Θ is the smaller angle between A and B, then

$\vec{A} . \vec{B} = AB\ Cos\Theta$

(i) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

(ii) $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

(iii) If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

Then $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Cross or Vector product :-

The cross product of two vectors $\vec{A}$ and $\vec{B}$, represented by $\vec{A} \times \vec{B}$ is a vector , which is equal to the product of the magnitudes of A and B and the sine of the smaller angle between them.

If Θ is the smaller angle between A and B, then

$\vec{A} \times \vec{B} = AB\ Sin\theta\ \hat{n}$

Where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

(i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

(ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}$ $\quad \hat{\jmath} \times \hat{k} = \hat{I}$ $\quad \hat{k} \times \hat{\imath} = \hat{\jmath}$

$\hat{\jmath} \times \hat{\imath} = -\hat{k}$ $\quad \hat{k} \times \hat{\jmath} = -\hat{\imath}$ $\quad \hat{\imath} \times \hat{k} = -\hat{\jmath}$

(iii) If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = (A_xB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

Projectile motion : - Projectile is the name given to anybody which once thrown in to space with some initial velocity, moves thereafter under the influence of gravity alone without being propelled by any engine or fuel. The path followed by a projectile is called its trajectory.

- Path followed by the projectile is parabola.
- Velocity of projectile at any instant t ,

$$V = [(u^2 - 2ugt\sin\theta + g^2t^2)]^{1/2}$$

- Horizontal range

$$R = u^2 \sin 2\Theta/g$$

For maximum range $\Theta = 45^0$,

$$R_{max} = u^2/g$$

- Flight time

$$T = 2u \sin\Theta/g$$

- Height

$$H = u^2 \sin^2\Theta/2g$$

For maximum height $\Theta = 90^0$

$$H_{max.} = u^2/2g$$

Very Short answer type questions (1 marks)

Q1. What does the slope of v-t graph indicate ?

Ans : Acceleration

Q2. Under what condition the average velocity equal to instantaneous velocity?

Ans :For a uniform velocity.

Q.3. The position coordinate of a moving particle is given by $x=6+18t+9t^2$ (x in meter, t in seconds) what is it's velocity at t=2s

Ans : 54 m/sec.

Q4. Give an example when a body moving with uniform speed has acceleration.
Ans : In the uniform circular motion.

Q5. Two balls of different masses are thrown vertically upward with same initial velocity. Height attained by them are h_1 and h_2 respectively what is h_1/h_2.

Ans : 1/1, because the height attained by the projectile is not depend on the masses.

Q6. State the essential condition for the addition of the vector.

Ans : They must represent the physical quantities of same nature.

Q7. What is the angle between velocity and acceleration at the peak point of the projectile motion ?

Ans : 90^0.

Q8. What is the angular velocity of the hour hand of a clock ?

Ans : $W = 2\pi/12 = \pi/6$ rad h^{-1},

Q9. What is the source of centripetal acceleration for earth to go round the sun ?

Ans. Gravitation force of the sun.

Q10. What is the average value of acceleration vector in uniform circular motion .

Ans : Null vector .

Short Answer type question (2 marks)

Q1. Derive an equation for the distance travelled by an uniform acceleration body in n^{th} second of its motion.

Ans. $S_n = u + \frac{a}{2}(2n - 1)$

Q2. The velocity of a moving particle is given by $V=6+18t+9t^2$ (x in meter, t in seconds) what is it's acceleration at t=2s

Ans. Differentiation of the given equation eq. w.r.t. time

We get $a = 18 + 18t$

At $t = 2$ sec.

$a = 54$ m/sec^2.

Q3.what is relative velocity in one dimension, if V_A and V_B are the velocities of the body A and B respectively then prove that $V_{AB}=V_A-V_B$?

Ans. Relative Motion:- The rate of change of separation between the two object is called relative velocity. The relative velocity of an object B with respect to the object A when both are in motion is the rate of change of position of object B with respect to the object A .

*Relative velocity of object A with respect to object B

$V_{AB} = V_A - V_B$

When both objects are moving in same direction , then the relative velocity of object B with respect to the object A

$V_{BA} = V_B - V_A$

Q4. Show that when the horizontal range is maximum, height attained by the body is one fourth the maximum range in the projectile motion.

Ans : We know that the horizontal range

$$R = u^2 \sin 2\Theta / g$$

For maximum range $\Theta = 45^0$,

$$R_{max} = u^2 / g$$

and Height

$$H = u^2 \sin^2\Theta / 2g$$

For $\Theta = 45^0$

$$H = u^2 / 4g = 1/4 \text{ of the } R_{max.}$$

Q6. State the parallelogram law of vector addition. Derive an expression for magnitude and direction of resultant of the two vectors.

Ans. The addition of two vector $\vec{A}$ and $\vec{B}$ is resultant $\vec{R}$

$$\vec{R} = \vec{A} + \vec{B}$$

And $R = (A^2 + B^2 + 2AB \cos\Theta)^{1/2}$

And $\tan \beta = B \sin\Theta / (A + B \cos\Theta)$,

Where Θ is the angle between vector $\vec{A}$ and vector $\vec{B}$, And β is the angle which $\vec{R}$ makes with the direction of $\vec{A}$.

Q7. A gunman always keeps his gun slightly tilted above the line of sight while shooting. Why,

Ans. Because bullet follow parabolic trajectory under constant downward acceleration.

Q8. Derive the relation between linear velocity and angular velocity.

Ans : Derive the expression

$$V = r \omega$$

Q9. What do you mean by rectangular components of a vector? Explain how a vector can be resolved into two rectangular components in a plane .

Q10. The greatest height to which a man can a stone is h, what will be the longest distance upto which he can throw the stone ?

Ans: we know that

$$H_{max.} = R_{max} /2$$

So $h = R/2$

Or $R = 2h$

Short answer questions (3 marks)

Q1. If 'R' is the horizontal range for Θ inclination and H is the height reached by the projectile, show that R(max.) is given by

$$R_{max} = 4H$$

Q2. A body is projected at an angle Θ with the horizontal. Derive an expression for its horizontal range. Show that there are two angles Θ_1 and Θ_2 projections for the same horizontal range. Such that $(\Theta_1 + \Theta_2) = 90^{0.}$

Q3. Prove that there are two values of time for which a projectile is at the same height . Also show that the sum of these two times is equal to the time of flight.

Q4: Draw position –time graphs of two objects , A and B moving along straight line, when their relative velocity is zero.

(i) Zero

Q5. Two vectors **A** and **B** are inclined to each other at an angle Θ. Using triangle law of vector addition, find the magnitude and direction of their resultant.

Q6. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a particle moving with constant speed v along a circular path of radius r.

Q7. When the angle between two vectors of equal magnitudes is 2π/3, prove that the magnitude of the resultant is equal to either.

Q8. A ball thrown vertically upwards with a speed of 19.6 m/s from the top of a tower returns to the earth in 6s. find the height of the tower. (g = 9.8 m/sec^2)

Q9. Find the value of λ so that the vector $\vec{A} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $\vec{B} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ are perpendicular to each.

Q10. Show that a given gun will shoot three times as high when elevated at angle of 60^0 as when fired at angle of 30^0 but will carry the same distance on a horizontal plane.

Long answer question (5 marks)

Q1. Draw velocity- time graph of uniformly accelerated motion in one dimension. From the velocity – time graph of uniform accelerated motion, deduce the equations of motion in distance and time.

Q2. (a) With the help of a simple case of an object moving with a constant velocity show that the area under velocity – time curve represents over a given time interval.

(b) A car moving with a speed of 126 km/h is brought to a stop within a distance of 200m. calculate the retardation of the car and the time required to stop it.

Q3. Establish the following vector inequalities :

(i) $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$

(ii) $|\vec{a} - \vec{b}| \leq |\vec{a}| + |\vec{b}|$

When does the equality sign apply.

Q4. What is a projectile ? show that its path is parabolic. Also find the expression for :

(i) Maximum height attained and

(ii) Time of flight

Q5. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r. explain how it acts along the radius towards the centre of the circular path.

HOTS

Q1. $\vec{A}$ and $\vec{B}$ are two vectors and Θ is the angle between them, If

$|\vec{A} \times \vec{B}| = \sqrt{3}\,(\vec{A}.\vec{B})$, calculate the value of angle Θ .

Ans : 60^0

Q2. A boat is sent across a river with a velocity of 8km/h. if the resultant velocity of boat is 10 km/h , then calculate the velocity of the river.

Ans : 6 km/h.

Q3. A cricket ball is hit at 45^0 to the horizontal with a kinetic energy E. calculate the kinetic energy at the highest point.

Ans : E/2.(because the horizontal component $u\cos 45^0$ is present on highest point.)

Q4. Speed of two identical cars are u and 4u at a specific instant. The ratio of the respective distances at which the two cars stopped from that instant.

Ans : 1 : 16

Q5. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the time of flight in the two cases, then prove that $t_1 t_2 = 2R/g$

ans : for equal range the particle should either be projected at an angle Θ and (90 - Θ) ,

$$\text{then} \qquad t_1 = 2u \sin\Theta/g$$

$$t_2 = 2u \sin(90 - \Theta)/g = 2u \cos\Theta/g$$

$$t_1 t_2 = 2R/g \ .$$