

# Key Notes

## Chapter-02

### Inverse Trigonometric Functions

- The domains and ranges (principal value branches) of inverse trigonometric functions are given in the following table:

Functions	Domain	Range (Principal Value Branches)
$y = \sin^{-1}x$	$[-1, 1]$	$\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$
$y = \cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$y = \operatorname{cosec}^{-1}x$	$\mathbb{R} - [-1, 1]$	$\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] - \{0\}$
$y = \sec^{-1}x$	$\mathbb{R} - [-1, 1]$	$\pi - \frac{\pi}{2}$
$y = \tan^{-1}x$	$\mathbb{R}$	$\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$
$y = \cot^{-1}x$	$\mathbb{R}$	$[0, \pi]$

$\sin^{-1}x$  should not be confused with  $(\sin x)^{-1}$ . In fact  $(\sin x)^{-1} = \frac{1}{\sin x}$  And similarly for other trigonometric functions

- The value of an inverse trigonometric functions which lies in its principal value branch is called the principal value of that inverse trigonometric functions.

**For suitable values of domain, we have**

- $y = \sin^{-1}x \Rightarrow x = \sin y$
- $x = \sin y \Rightarrow y = \sin^{-1}x$
- $\sin(\sin^{-1}x) = x$
- $\sin^{-1}(\sin x) = x$

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- $\sin^{-1} \frac{1}{x} = \csc^{-1} x$
- $\cos^{-1} (-x) = \pi - \cos^{-1} x$
- $\cos^{-1} \frac{1}{x} = \sec^{-1} x$
- $\cot^{-1}(-x) = \pi - \cot^{-1} x$
- $\tan^{-1} \frac{1}{x} = \cot^{-1} x$
- $\sec^{-1}(-x) = \pi - \sec^{-1} x$
- $\sin^{-1} (-x) = -\sin^{-1} x$
- $\tan^{-1} (-x) = -\tan^{-1} x$
- $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
- $\operatorname{cosec}^{-1} (-x) = -\operatorname{cosec}^{-1} x$
- $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$
- $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
- $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$
- $2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}$
- $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$
- $2 \tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2} = \frac{1-x^2}{1+x^2}$