

Key Notes

Chapter-10

Vector Algebra

- Position vector of a point P (x, y, z) is given as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and its magnitude by $\sqrt{x^2 + y^2 + z^2}$
- The scalar components of a vector are its direction ratios, and represent its projections along the respective axes.
- The magnitude (r), direction ratios (a, b, c) and direction cosines (l, m, n) of any vector are related as: $l = \frac{a}{r}, m = \frac{b}{r}, n = \frac{c}{r}$
- The vector sum of the three sides of a triangle taken in order is $\vec{0}$
- The vector sum of two conical vectors is given by the diagonal of the parallelogram whose adjacent sides are the given vectors.
- The multiplication of a given vector by a scalar λ , changes the magnitude of the vector by the multiple $|\lambda|$, and keeps the direction same (or makes it opposite) according as the value of λ is positive (or negative).
- For a given vector \vec{a} , the vector $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$ gives the unit vector in the direction of \vec{a}
- The position vector of a point R dividing a line segment joining the points P and Q whose position vectors are \vec{a} and \vec{b} respectively, in the ratio $m : n$
 - (i) internally, is given by $\frac{m\vec{b} + n\vec{a}}{m+n}$
 - (ii) externally, is given by $\frac{m\vec{b} - n\vec{a}}{m-n}$
- The scalar product of two given vectors \vec{a} and \vec{b} having angle θ between them is defined as $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$

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Also, when $\vec{a} \cdot \vec{b}$ is given, the angle θ between the vectors \vec{a} and \vec{b} may be determined by

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

- If θ is the angle between two vector \vec{a} and \vec{b} , then their cross product is given as

$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin\theta \hat{n}$ where \hat{n} is a unit vector perpendicular to the plane containing \vec{a} and \vec{b} .

Such that \vec{a} , \vec{b} , \hat{n} form right handed system of coordinate axes.

- If we have two vectors \vec{a} and \vec{b} given in component form as $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and λ any scalar, then, $\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$

$$\lambda\vec{a} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k};$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\text{and } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$