## Chapter-4

## Determinant

- Determinant of a matrix $A = [a_{11}]_{1\times 1}$ is given by $|a_{11}| = a_{11}$

- Determinant of a matrix A $\begin{matrix} a_{11} & 12 \\ a_{21} & a_{22} \end{matrix}$ is given by

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\ a_{22} - a_{12}\ a_{21}$$

- Determinant of a matrix A $\begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix}$ is given by (expanding along ($R_1$)

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

- **For any square matrix A, the |A| satisfy following properties.**

- $|A'| = |A|$, where $A'$ = transpose of A.

- If we interchange any two rows (or columns), then sign of determinant changes.

- If any two rows or any two columns are identical or proportional, then value of determinant is zero.

- If we multiply each element of a row or a column of a determinant by constant k, then value of determinant is multiplied by k.

- Multiplying a determinant by k means multiply elements of only one row (or one column) by k.

- *If* $A = \left[ a_{ij} \right]_{3\times 3}$ *, then* $|k.\ A| = k^3 |A|$

- If elements of a row or a column in a determinant can be expressed as sum of two or more elements, then the given determinant can be expressed as sum of two or more determinants.

- If to each element of a row or a column of a determinant the equimultiples of corresponding elements of other rows or columns are added, then value of determinant remains same.

- Area of a triangle with vertices $(x_1, y_1)$, $(x_2, y_2)$ $and$ $(x_3, y_3)$ is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

- Minor of an element aij of the determinant of matrix A is the determinant obtained by deleting $i^{th}$ row and $j^{th}$ column and denoted by $M_{ij}$

- Cofactor of aij of given by Aij = (– 1)i+ j Mij

- Value of determinant of a matrix A is obtained by sum of product of elements of a row (or a column) with corresponding cofactors. For example, $/A/ = a_{11} A_{11} + a_{12} A_{12} + a_{13}A_{13}$.

- If elements of one row (or column) are multiplied with cofactors of elements of any other row (or column), then their sum is zero. For example, $_{11}$ $_{21}$ $+ a_{12}A_{22} + a_{13} A_{23} = 0$

- $A\left(adj\ A\right) = \left(adj\ A\right) A = \left|A\right| I,$ where A is square matrix of order n.

- A square matrix A is said to be singular or non-singular according as $\left|\ \right\| = 0\ or\ \left|A\right| \neq 0.$

- If AB = BA = I, where B is square matrix, then B is called inverse of A. Also $A^{-1} = B\ or\ B^{-1} = A\ and\ hence\ \left(A^{-1}\right)^{-1} = A.$

- A square matrix A has inverse if and only if A is non-singular.

- $A^{-1} = \frac{1}{|A|}(adj\ A)$

- If $a_1x + b_1y + c_1z = d_1$

- $a_2x + b_2y + c_2z = d_2$

- $a_3x + b_3y + c_3z = d_3$

- then these equations can be written as A X = B, where

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$$A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = X \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

- Unique solution of equation AX = B is given by $X = A^{-1}B, \; where \; |A|/0 \neq 0$.

- A system of equation is consistent or inconsistent according as its solution exists or not.

- For a square matrix A in matrix equation AX = B

- $|{}^{/A}| \neq 0,$ there exists unique solution

- |A| = 0 and (adj A) B ≠ 0, then there exists no solution

- $|A| = 0 \; and \; (adj \, A) \, B = 0,$ then system may or may not be consistent.