Chapter-5

Continuity and Differentiability

- A real valued function is continuous at a point in its domain if the limit of the function at that point equals the value of the function at that point. A function is continuous if it is continuous on the whole of its domain.
- Sum, difference, product and quotient of continuous functions are continuous. i.e., if f and g are continuous functions, then

  $(f \pm g)(x) = f(x) \pm g(x)$ is continuous.

  $(f \cdot g)(x) = f(x) \cdot g(x)$ is continuous.

  $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ (wherever $g(x) \neq 0$) is continuous.

- Every differentiable function is continuous, but the converse is not true.
- Chain rule is rule to differentiate composites of functions. If $f = v \circ u$, $t = u(x)$ and if both *and if both* $\frac{dt}{dx}$ *and* $\frac{dv}{dt}$ exist then $\frac{df}{dx} = \frac{dv}{dt} = \frac{dt}{dx}$

- Following are some of the standard derivatives (in appropriate domains):

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\operatorname{cosec}^{-1}x\right)=\frac{-1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(e^{x}\right)=e^{x}$$

$$\frac{d}{dx}(\log x)= 1$$

- Logarithmic differentiation is a powerful technique to differentiate functions of the form $f(x) = [u(x)]^{v(x)}$ Here both f (x) and u (x) need to be positive for this technique to make sense.
- **Rolle's Theorem:** If f: [a, b] → R is continuous on [a, b] and differentiable on (a, b) such that f (a) = f (b), then there exists some c in (a, b) such that f ′(c) = 0.
- Mean Value Theorem: If f: [a, b] → R is continuous on [a, b] and differentiable on (a, b). Then there exists some c in (a, b) such that $f'(c)=\frac{f(b)-f(a)}{b-a}$